which has a unique solution achievable by (1.20).

It follows from Theorems 1-4 that in all the problems considered the sets of creep problem solutions (including the solutions of the elasticity problems as special cases) are lineals of finite dimensionality for all possible values of the rheological characteristics. Consequently, the available arbitrariness in selecting the constants G_k, ν_k of the auxiliary elasticity problems essentially denotes the possibility of selecting different bases in this lineal.

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A METHOD FOR THE AUTOMATIC EXTINCTION OF DIRECTIONAL FORCES BY MEANS OF BALL SELFBALANCERS*

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We consider the possible extinction of a directional harmonic force by means of two like selfbalacing systems (SBS) leading to rotation in two opposite directions with a frequency equal to the frequency of the acting force. A method of extinguishing circulating forces caused by rotor imperfections by means of ball SBS was described in /1/. The action of directional forces, e.g., forces due to the operation of crank-and-rod mechanisms, is usually extinguished by means of a system of two constant unbalancers rotating in opposite directions. The latter have poor efficiency, however, if the amplitude or direction of the acting force can vary in time. In this case it is best to use a system of two unbalancers, whose values vary in accordance with the variation of the external disturbing force.

The dynamic characteristics of our theoretical model (Fig.1) will be assumed to be the same in all directions at the location of the selfbalancers and to be given as an impedance ξ_c. Let a directional harmonic force $F = 2D\omega^2 \cos(\omega t + \eta_0)$ act on the system at an angle φ_0.

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Notice that the action of the force F is equivalent to the action of two unbalancers $D_1 = D_2 = D$, rotating in opposite directions symmetrically about the direction given by the angle φ_0. In the general case $D_1 \neq D_2$ the action of these unbalancers is equivalent to the action of a directional force with amplitude $2D_2\omega^2$ (or $2D_1\omega^2$) and an unbalancer $|D_1 - D_2|$, rotating clockwise (if $D_2 > D_1$) or counterclockwise (if $D_1 > D_2$).

The equations of motion of the balls are /2/

$$\varphi_i^{\cdot\cdot} + \beta [\varphi_i^{\cdot} + (-1)^n \omega] = \mu\Phi(\varphi_i, x, y) \qquad (1)$$
$$\mu\Phi = R^{-1} [x^{\cdot\cdot} \sin\varphi_i - y^{\cdot\cdot} \cos\varphi_i]$$

Here and below, the subscript values $n = 1$ and $i = 1, \ldots, N$, refer to the balls which rotate counter-clockwise, and $n = 2$ and $i = N + 1, \ldots, 2N$ to those rotating clockwise, μ is a small parameter, φ_i is the angle giving the position of the i-th ball in the SBS cage, N is the number of balls in each selfbalancer, x and y are the coordinates of the deviation of point O of the system from the equilibrium position, β is the coefficient of viscous friction in the relative motion of the balls, and R is the radius of the races of the selfbalancers. The equations of the oscillations of point O of the system can be written as /1/

$$L\begin{Bmatrix} x \\ y \end{Bmatrix} = H\left[D_1\omega^2 \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}(\omega t + \eta_0 + \varphi_0) + D_2\omega^2 \begin{Bmatrix} \cos \\ -\sin \end{Bmatrix}(\omega t + \eta_0 - \varphi_0) - \right. \qquad (2)$$
$$\left. 2mN\begin{Bmatrix} x^{\cdot\cdot} \\ y^{\cdot\cdot} \end{Bmatrix} + mR\sum_{i=1}^{2N}\left(\varphi_i^{\cdot\cdot}\begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}\varphi_i + \varphi_i^{\cdot 2}\begin{Bmatrix} \cos \\ \sin \end{Bmatrix}\varphi_i\right)\right]$$

where m is the mass of one ball, and L and H are linear differential operators, defining the impedance ξ_c.

By the principles of synchronization of dynamic systems /2/, the generating system of equations $(\mu \equiv 0)$ corresponding to (1) and (2) has the solution

$$\varphi_i^{\circ} = (-1)^{n+1}\omega t + \alpha_i$$
$$x_0(t) = q[(D_{x1} + D_{x2})\cos(\omega t + \lambda) - (D_{y1} - D_{y2})\sin(\omega t + \lambda)] \qquad (3)$$
$$y_0(t) = q[(D_{x1} - D_{x2})\sin(\omega t + \lambda) + (D_{y1} + D_{y2})\cos(\omega t + \lambda)]$$
$$-j\omega/(\xi_c + j2\omega mN) = qe^{j\lambda}, \quad j = \sqrt{-1}$$

$$\begin{Bmatrix} D_{x1} \\ D_{y1} \end{Bmatrix} = D_1\begin{Bmatrix} \cos \\ \sin \end{Bmatrix}\chi_1 + mR\sum_{i=1}^{N}\begin{Bmatrix} \cos \\ \sin \end{Bmatrix}\alpha_i, \quad \chi_1 = \varphi_0 + \eta_0$$
$$\begin{Bmatrix} D_{x2} \\ D_{y2} \end{Bmatrix} = D_2\begin{Bmatrix} \cos \\ \sin \end{Bmatrix}\chi_2 + mR\sum_{i=N}^{2N}\begin{Bmatrix} \cos \\ \sin \end{Bmatrix}\alpha_i, \quad \chi_2 = \varphi_0 - \eta_0$$

D_{x1} and D_{y1} are the projections of the total unbalance vector of the system, rotating counter-clockwise, and D_{x2} and D_{y2} are the same in the clockwise case.

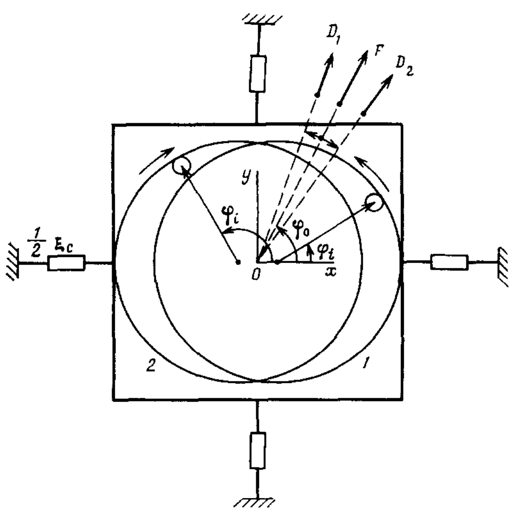

Fig.1

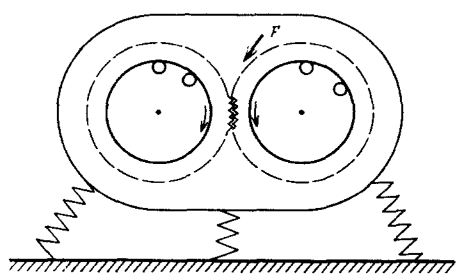

Fig.2

The values of the generating phases α_i are found by equating to zero the generating functions P_i, where

$$\mu P_i = \frac{2}{\beta} \int_0^{2\pi/\omega} \mu\Phi(\varphi_i^\circ, x_0, y_0)\,dt, \quad i = 1, 2, \ldots, 2N$$

whence we obtain

$$\mu P_i = -\frac{4\pi\omega v}{\beta R} \{D_{xn} \sin[\alpha_i + (-1)^n \lambda] - D_{yn} \cos[\alpha_i + (-1)^n \lambda]\} \tag{4}$$

The system of $2N$ equations $\mu P_i = 0$ for the important practical case of two balls in each selfbalancer ($N = 2$) gives 16 different solutions for the positions of the balls in the selfbalancers, of which the only solution corresponding to the absence of oscillations of the system refers to the case $D_{xn} = D_{yn} = 0$, or

$$\begin{aligned} &\alpha_1 - \chi_1 = -(\alpha_2 - \chi_1), \ \cos(\alpha_1 - \chi_1) = -D_1/(2mR) \\ &\alpha_3 - \chi_2 = -(\alpha_4 - \chi_2), \ \cos(\alpha_3 - \chi_2) = -D_2/(2mR) \end{aligned} \tag{5}$$

It can be seen from (5) that each selfbalancer compensates only the component of force F which rotates in the same direction as the corresponding SBS cage.

The common condition that the equilibrium positions of the balls be asymptotically stable is that the real parts of the roots of the fourth degree equation in z /2/

$$\det(\partial P_i/\partial\alpha_k - \delta_{ik}z) = 0; \ i, k = 1, \ldots, 4 \tag{6}$$

be negative. We can show by the Routh-Hurwitz method that the position of the balls given by (5) under the condition $D_n < 2mR$ is the only stable position in the range of rotation frequencies defined by the criterion

$$\operatorname{Im}\{\xi_c + j2\omega mN\} > 0 \tag{7}$$

which is the same as the criterion obtained in /1/ for the case of a single selfbalancer for extinguishing vibrations due to unbalance of a rotating rotor.

This conclusion was checked experimentally for the elementary case of a system consisting of a body on an isotropic elastic suspension (Fig.2) with natural frequency of oscillation ω_0; condition (7) corresponds to the inequality $\omega > \omega_0$.

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CONTROL OF THE SPECTRUM OF MULTIDIMENSIONAL OSCILLATORY OBJECTS*

V.A. BRUSIN

The solution is obtained in closed form of the problem of shifting in the complex plane any pairs by simple complex conjugate eigenvectors of the linear part of a controlled object by means of linear output variable feedbacks.

Consider the linear controlled object described by the equations

$$x^{\cdot} = Ax + Bu, \quad y = Cx \tag{1}$$

where x is the n-dimensional state vector, y is the m-dimensional output signal vector, u is the r-dimensional control vector, and A, B, C are $n \times n$, $n \times r$, and $m \times n$ matrices respectively.

A problem in stability and control theory concerns the control of the spectrum of a

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